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The Sward Number and Mechanical Properties of Plastics

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Synopsis

A Sward rocker tester is used to obtain the Sward number for glass, mild steel, copper, poly(methyl methacrylate), polyethylene of various densities, and natural rubber. A relationship between the number and mechanical properties is investigated. It is shown that with metals and glass the number is essentially a frictional factor. With plastics and rubber it is a true hardness factor, involving dynamic Young's modulus, Poisson's ratio, and damping capacity. The aim of the investigation is to encourage the development of the Sward test as a nondestructive quality test for plastics.

Introduction

The rocker test was originally devised by Sward[1] as a hardness test for paint films. The instrument is a pair of circular rockers or flat metal rings, linked in parallel, with a pendulum pivoted at the top between the rockers. As the rocker oscillates from side to side on a flat surface the pendulum indicates the amplitude of oscillation. The number of pendulum oscillations n between two reference points is multiplied by two to give the Sward number, $N = 2n$, for the material tested.

The simplicity of operation gives the instrument considerable practical advantage as a nondestructive test. Perhaps the test has not been considered for plastics because the relationship between N and dynamical mechanical properties has not been investigated. The first step is thus the development of an approximate quantitative relationship.

Theoretical

Parker and Siddle's theory for the pendulum tests which preceded Sward indicates that a simplified analysis based on the use of energy equations is the best approach.[2] In Figure 1 the rocker is shown in its extreme position, where θ, the displacement angle, has its maximum value θ_1. The rocker also has its maximum potential energy. Denoting the value of θ at the beginning of the second oscillation by θ_2, then ΔU, the energy lost during the first oscillation is:

$$\Delta U = Mga\,(\cos \theta_2 - \cos \theta_1) \quad (1)$$

where M is the mass of the rocker, g the gravitational constant, and a the distance between the center of gravity and the geometrical center of the rocker.

The Sheen[3] form of the rocker requires that the number of oscillations between $\theta = 0.58$ radians (33°) and $\theta = 0.28$ radians (16°) be counted to give the Sward number. Assuming an approximately linear decrease in amplitude over this amplitude range, then:

$$N = 0.60/(\theta_1 - \theta_2) \tag{2}$$

If the term $(\theta_1 - \theta_2)$ is small, then the cosine difference in eq. (1) can be approximated to $(\theta_1 - \theta_2)\sin\theta_1$, since $(\theta_1 + \theta_2)/2 \sim \theta_1$ and $\sin(\theta_1 - \theta_2)/2 \sim (\theta_1 - \theta_2)/2$. Combining eqs. (1) and (2) gives

$$N = 0.60\ Mga \sin\theta_1/\Delta U \tag{3}$$

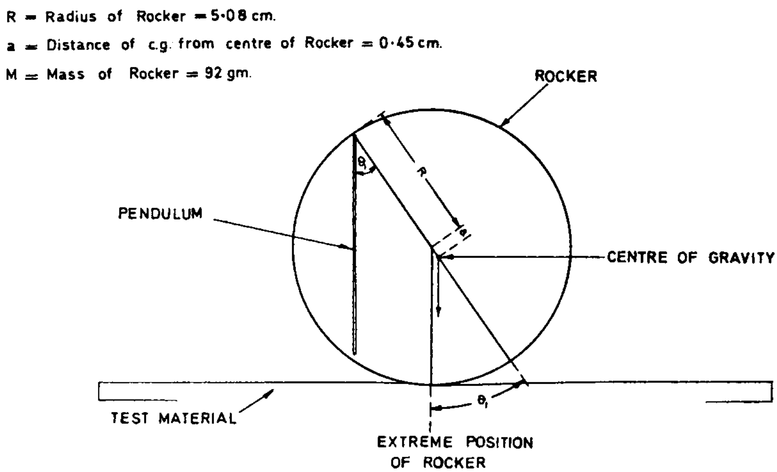

Fig. 1. Dimensions of rocker.

The total energy loss ΔU is divided into its two components, the surface friction loss ΔU_s, and the internal friction loss ΔU_i.

The surface loss is due to rolling friction. The coefficient of rolling friction λ is, in Shaw's notation,[4] defined by $T_r = \lambda W$, where T_r is the force applied to the center of a sphere to roll it without slip, and W is the weight of the sphere. If the rocker travels a distance D in one oscillation, then $\Delta U_s = \lambda WD$. In the first oscillation $D \sim 2(\theta_1 + \theta_2)R$, where R is the radius of the rocker rings. Thus $\Delta U_s = 2\lambda MgR(\theta_1 + \theta_2)$. Friction data are usually given in terms of the sliding friction coefficient μ. Shaw's paper[4] gives the ratio λ/μ for a sphere, radius R, rolling on a flat surface as $l\sqrt{d/2kR}$, where d is the distance between atoms and kd the distance bonds stretch before rupture during the rolling process. Similarly ld is the distance for bond rupture in the sliding friction process when a sphere is drawn by a force T_s, without rotation, over a flat surface. In this case $T_s = \mu W$. Putting $\alpha = l\sqrt{d/2k}$ as a molecular constant and $(\theta_1 + \theta_2) \sim 2\theta_1$ gives:

$$\Delta U_s = 4\alpha\mu Mg\theta_1\sqrt{R} \tag{4}$$

The internal friction loss arises from the dynamic indentation of the test material as the rocker rolls over the surface. The stored energy of deformation for one indentation when the strain is a maximum is $e = Mgx/2$, where x is the depth of indentation. The specific damping capacity S gives the energy loss Δe, corresponding to e, in the form $S = 2\Delta e/e$, because the rocker only takes the test material through the compressive half of a full stress cycle.[5] Denoting the diameter of the indentation along the rolling direction by w, then the number of indentations occurring in one oscillation period for one rocker ring is $2R(\theta_1 + \theta_2)/w$. If $\theta_1 \sim \theta_2$, then the total loss for both rings is:

$$\Delta U_t = 2SMgR\theta_1\,(x/w) \tag{5}$$

Relating the ratio x/w to mechanical properties appears difficult with the indentation involving both the orthogonal radii: the ring radius R, and the radius, 0.5 mm., of the curved surface on the outer edge of each ring. However, data given by Bosco[6] suggest that the Sward number is insensitive to variations of this latter radius, and it is therefore possible to use the elastic indentation theory[7] for a sphere radius R and a flat surface. Thus $w = 2\sqrt{xR}$ and $x^{3/2} = \text{Mg}/2q$, where $q = 4\sqrt{R}/3\pi(\beta_R + \beta_T)$. In the factor $\beta_R = (1 - \nu_R^2)/\pi E_R$, the Poisson's ratio and Young's modulus of the rocker ring material are denoted by ν_R and E_R, respectively. The factor β_T is the corresponding quantity for the test material.

Combining the above energy loss results in eq. (3) gives:

$$1/N = A\alpha\mu + BS(\beta_R + \beta_T)^{1/3} \tag{6}$$

where A and B are instrument constants:

$$A = 4\theta_1\sqrt{R}/0.6\ a \sin\theta_1 \tag{7}$$

$$B = R\theta_1(3\pi Mg/R^2)^{1/3}/1.2\ a \sin\theta_1$$

If the rate of deformation is required it must be estimated for each material from $4R\theta_1/w$.

When the damping is low, the damping capacity S is approximately proportional to the loss factors appearing in other descriptions of internal friction.[5] If the modulus E is written in terms of its real and imaginary parts, E_1 and E_2, so that $E = E_1 + iE_2$ and Δ' denotes the logarithmic decrement, then $0.5S = \pi E_2/E_1 = \Delta' = \pi\delta$, where δ is the ratio E_2/E_1.

The apparatus to which the above theory is applied is now described.

Apparatus

The rocker used[3] is shown diagrammatically in Figure 2. It is essentially a pair of circular flat chromium plated brass rings of outer radius $R = 2$

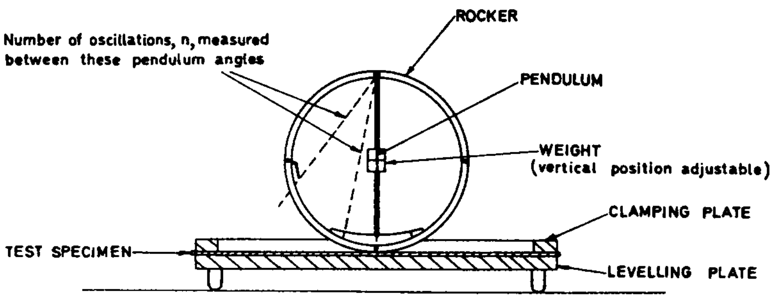

Fig. 2. Apparatus.

in., spaced 1 in. apart. The outer edge of each ring is machined to 0.5 mm. radius. At the top of the rocker, a pendulum is mounted on pivot bearings between the rings and indicates the angle of oscillation of the rocker. The lower end of the pendulum terminates in a pointer which swings across a scale as the rocker oscillates. A weight is threaded on a vertical spindle as shown in Figure 2, the position of the weight along the spindle being adjusted to obtain the required period of oscillation.

The test specimens used here are small (approximately 4 in. × 6 in.) clean flat sheets. A specimen is clamped on to a leveling plate which is so adjusted that the test surface is horizontal. Tests are carried out in a draft-free enclosure.

The rocker is placed on the test surface and is tilted to the left until the pendulum touches a stop. It is then allowed to rock from side to side and the decay of the amplitude of the oscillation observed. The number of oscillations n between the last time the pendulum touches the stop for $\theta = 0.58$ radians (33°) and when it last reaches the mark on the scale for $\theta = 0.28$ radians (16°) is measured, and multiplied by two to give the Sward number.

The rocker is initially brought to a standard setting on a clean unscratched sheet of plate glass by adjusting the spindle weight to give an oscillation period of two seconds. This corresponds to $n = 50$ and $a = 0.45$ cm. The cleanliness of the contact surfaces and the adjustment of the rocker period is checked frequently by reaffirming that $n = 50$ on the standard glass plate.

The preparation of test surfaces involves the washing of specimens in soapy water. They are then rinsed, cleaned with a solvent, and finally rinsed in distilled water and dried.

The apparatus constants of eq. (7) are calculated to be $A = 35.6$ cm.$^{-1/2}$ and $B = 321$ (g. cm.$^{-1}$ sec.$^{-12}$)$^{1/2}$ for $\theta = \theta_1 = 0.58$ radians.

Experimental Results

For each specimen tested, a set of readings consists of five consecutive measurements of n, with a check on the standard glass surface before and afterwards.

TABLE I
Experimental Results and Published Data

Material	Density, g./cm.^{3}	Sward number N	Philip's roughness, ru	Dynamic modulus: $E \times 10^{-11}$ dyne/cm.^{2}	Dynamic modulus: Test frequency, cps	Poisson's ratio ν	Damping capacity S	$\beta_T \times 10^{12}$, cm.^{2}/dyne[a]
Glass	2.5	100	1	70	—	0.25	$>3 \times 10^{-6}$	0.43
Mild Steel	7.8	93	2	21	—	0.29	10^{-4}–10^{-3}	0.14
Copper	8.9	80	10	12	—	0.34	10^{-4}–10^{-3}	0.23
PMMA	1.19	32	1	0.46	40	0.35	0.44	6.07
Polyethylene 1	0.953	24	10	0.21	42	0.42	0.15	12.50
Polyethylene 2	0.951	18	30	0.15	38	0.42	0.24	17.50
Polyethylene 3	0.938	21	10	0.11	33	0.43	0.40	23.6
Polyethylene 4	0.920	11	—	0.04	20	0.45	0.75	61.9
Natural rubber	1.14	7	—	0.0024	50	0.50	0.63	1000

[a] $\beta_R < 0.3 \times 10^{-12}$ cm.^{2}/dyne

To ensure the reproducibility of results, several sets of readings are obtained for each specimen. The Sward numbers N given in Table I are the simple average values of n multiplied by two. Also in Table I are the published dynamical mechanical data with which the values of N are to be correlated.

The dynamic moduli and damping capacities for the thermoplastic were obtained using a resonance test developed in this laboratory.[8] The data for rubber is from Snowdon's[9] paper. The damping capacity of glass is assumed to be a little greater than 3×10^{-6}, the figure for quartz,[10] and the values for the metals will be in the range 10^{-4}–10^{-3}.[10] The dynamic moduli and Poisson's ratios for glass and the metals are taken from Kolsky[5] and Kaye and Laby.

The rocker rings are of chromium-plated brass. Since the modulus of chromium is more than twice that of brass,[12] the maximum value of $\beta_R = 0.3 \times 10^{-12}$ cm.^{2}/dyne is given by the minimum modulus for brass (9×10^{11} dyne/cm.^{2})[11,12] and the average Poisson's ratio 0.37.[11]

The above data are now considered in terms of equation (6).

Discussion

Since the factors α and μ are both influenced by the surface condition of the test material, particularly with regard to roughness, a Philips roughness meter is used to give a comparative control. It is preferable for surfaces to have a low roughness value of less than 10 ru. An excessively rough surface gives a reading of the order 100 ru.

From the data of Table I the values of $1/N$ for glass, steel, and copper are found to be very much greater than the term $BS(\beta_R + \beta_T)^{1/3}$, so the Sward number is essentially a frictional factor given by $N = 1/A\alpha\mu$. This result can only be considered in more detail by taking the coefficients of friction for a curved steel surface, rather than chromium, sliding on a flat metallic surface. For steel $\mu = 1.0$, and for copper $\mu = 0.9$.[13] The data for the calculation of $\alpha = l\sqrt{d/2k}$ are available only for steel sliding on steel. Electrical conductance measurements for this case show rapid fluctuations corresponding to a relative motion of the surfaces of $ld \sim 10^{-6}$ cm.[13] Taking $l = k$ gives for steel $\alpha = 7 \times 10^{-4}$ cm.$^{1/2}$. From the data of Table I the experimental value of $\alpha = 1/NA\mu$ for steel is 3×10^{-4} cm.$^{1/2}$, and for copper 4×10^{-4} cm.$^{1/2}$. It should be noted that increasing roughness will increase ld and decrease N.

The application of eq. (6) to thermoplastics and rubber is difficult because α cannot be calculated. The experimental results must therefore be used to investigate the possibility that the term $A\alpha\mu$ is negligible when it is assumed that $\alpha = 3 \times 10^{-4}$ cm.$^{1/2}$ as for steel. If this is so, then the Sward number is likely to be useful in estimating damping capacity. Calculated values of S obtained from the two possible forms of eq. (6):

$$S = (1 - NA\alpha\mu)[\pi E/(1 - \nu^2)^{1/3}]^{1/3}/NB \tag{8}$$

and

$$S = [\pi E/(1 - \nu^2)]^{1/3}/NB \tag{9}$$

are compared with published values. In eq. (8) the coefficient of friction for PMMA is $\mu = 0.45$, for low-density polyethylene 0.2, and for rubber 1.0.[13] For the polyethylenes of high density it is assumed that $\mu \sim 0.2$.

The various values of S are given in Table II. For the materials with smooth surfaces, PMMA, polyethylenes 3 and 4, and rubber, eq. (9) gives a reasonably accurate estimate of S. For the polyethylenes 1 and 2 this is not so, probably because the test panels cut from commercial sheet cannot be clamped in a strain-free condition. However if the clamping conditions are reproducible the Sward number is still a useful comparative factor.

TABLE II
Damping Capacity of Nonmetals

Material	Damping capacity S		
	From eq. (9)	From eq. (10)	From published data[11]
PMMA	0.45	0.53	0.44
Polyethylene 1	0.53	0.56	0.15
Polyethylene 2	0.64	0.67	0.24
Polyethylene 3	0.49	0.52	0.40
Polyethylene 4	0.69	0.71	0.75
Natural rubber (50 phr black)	0.41	0.45	0.63[a]

[a] Data of Roberts and Steel.[14]

The rubber tested is a 50 parts carbon black per 100 parts natural rubber mix. The Sward value of S is compared with the ratio $\delta = 0.1$,[9] by $\delta = S/2\pi$.

Conclusions

The relationship between Sward number and frictional and dynamic mechanical properties derived in this investigation is only a first approximation. It does, for example, neglect the damping effect of the atmosphere. However, the experiments show that it does nevertheless provide a quantitative basis for the Sward test. For glass and metals the number is essentially a frictional factor, whilst for thermoplastics and rubber it is a true hardness factor.

For the latter materials the simplified eq. (9) $N = [\pi E/(1 - \nu^2)]^{1/3}/BS$, seems likely to be the most useful theoretical result. This only gives a reasonable indication of damping capacity if the test specimen is strain free and its surface smooth and not tacky. If these conditions are not met the Sward number can still be a useful comparative factor if the test conditions are reproducible.

Since a smooth surface is required for quantitative work the rocker should, whenever possible, be used in conjunction with the Philips roughness meter.

The Sward test will be most useful where damping capacity varies more than the other mechanical properties with changes in molecular structure or polymer additive. For example, preliminary experiments suggest that the method may be used to measure the degree of cure of glass fiber laminate resins.[14]

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Résumé

Un appareillage d'essai à oscillation de Sward a été utilisé pour obtenir l'indice Sward pour le verre, l'acier doux, le cuivre, le polyméthylméthacrylate (PMMA), le polyéthylène de diverses densités et le caoutchouc naturel. Le rapport entre cet indice et les propriétés mécaniques a été étudié. On montre qu'avec des métaux et le verre, cet indice est essentiellement l'expression d'un facteur de friction. Avec les plastiques et le caoutchouc, c'est un véritable facteur de dureté, comprenant le module dynamique de Young, le rapport de Poisson et la capacité d'amortissement. Le but de cette recherche est d'encourager le développement du test de Sward comme méthode qualitative non-destructive pour l'examen des plastiques.

Zusammenfassung

Ein Sward-Rocker-Tester wird zur Bestimmung der Sward-Zahl von Glas, schweissbarem Stahl, Kupfer, Polymethylmethacrylat (PMMA), Polyäthylen verschiedener Dichte und Naturkautschuk verwendet. Die Beziehung zwischen dieser Zahl und den mechanischen Eigenschaften wird untersucht. Es wird gezeigt, dass die Zahl bei Metallen und Glas im wesentlichen ein Reibungsfaktor ist. Bei Plastomeren und Kautschuk ist sie ein wirklicher, aus dem dynamischen Young-Modul, dem Poisson-Verhältnis und der Dämpfungskapazität zusammengesetzter Härtefaktor. Das Ziel der Arbeit ist die Förderung der Entwicklung des Sward-Tests als eines zerstörungsfreien Qualitätstests für Plastomere.

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